Chapter (2)

does not meet $b^2-4ac<0$

two point $b^2-4ac>0$

two equal $b^2-4ac=0$

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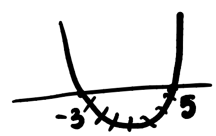
1. Find the values of k for which the line $y = kx - 3$ does not meet the curve $y = 2x^2 - 3x + k$. [5]

$$kx-3=2x^2-3x+k$$
$$0=2x^2-3x+k-kx+3$$

$$2x^2-3x+k-kx+3=0$$
$$2x^2-3x-kx+k+3=0$$
$$2x^2-3x-kx+k+3=0$$

$a=2,\ b=-3-k\ ,\ c=k+3$

$$b^2-4ac<0$$
$$(-3-k)^2-4(2)(k+3)<0$$
$$9+6k+k^2-8(k+3)<0$$
$$9+6k+k^2-8k-24<0$$
$$k^2-2k-15<0$$
$$(k-5)(k+3)<0$$



$$-3<k<5$$

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2. Solve the simultaneous equations

$$3x^2 - xy + 2y^2 = 16$$

$$2y - x = 4 \quad [5]$$

$-x = 4 - 2y$

$x = 2y - 4$

$3(2y-4)^2 - (2y-4)y + 2y^2 = 16$

$3(4y^2 - 16y + 16) - \cancel{2y^2} + 4y + \cancel{2y^2} - 16 = 0$

$12y^2 - 48y + 48 + 4y - 16 = 0$

$12y^2 - 44y + 32 = 0$

$(3y - 8)\quad(y - 1) = 0$

$y = \frac{8}{3}$ $\qquad$ $y = 1$

$x = 2y - 4$ $\qquad$ $x = 2y - 4$

$= \frac{16}{3} - 4$ $\qquad$ $= 2 - 4$

$= \frac{16 - 12}{3}$ $\qquad$ $= -2$

$= \frac{4}{3}$

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3. (a) Find the set of values of x for which $4x^2 + 19x - 5 \leq 0$. [3]

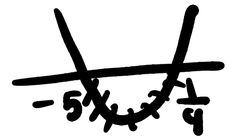
$x = \frac{1}{4}$

$x = -5$

$4x^2 + 19x - 5 \leq 0$

$(4x - 1)(x + 5) \leq 0$

$-5 \leq x \leq \frac{1}{4}$



(b) (i) Express $x^2 + 8x - 9$ in the form $(x + a)^2 + b$, where a and b are integers. [2]

$x^2 + 8x - 9 = (x + a)^2 + b$

$= x^2 + 2ax + a^2 + b$

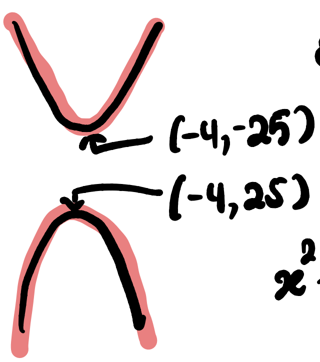
$8x = 2ax$

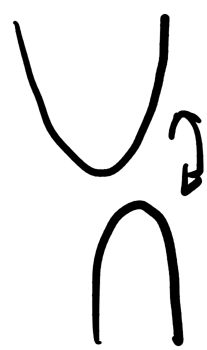
$a = 4$

$a^2 + b = -9$

$16 + b = -9$

$b = -25$





$x^2 + 8x - 9 = (x + 4)^2 - 25$

(ii) Use your answers to part (i) to find the greatest value of $9 - 8x - x^2$ and the value of x at which this occurs. [2]

Greatest value $= 25$

value of x $= -4$

(iii) Sketch the graph of $y = 9 - 8x - x^2$, indicating the coordinates of any point of intersection with the coordinate axes. [2]

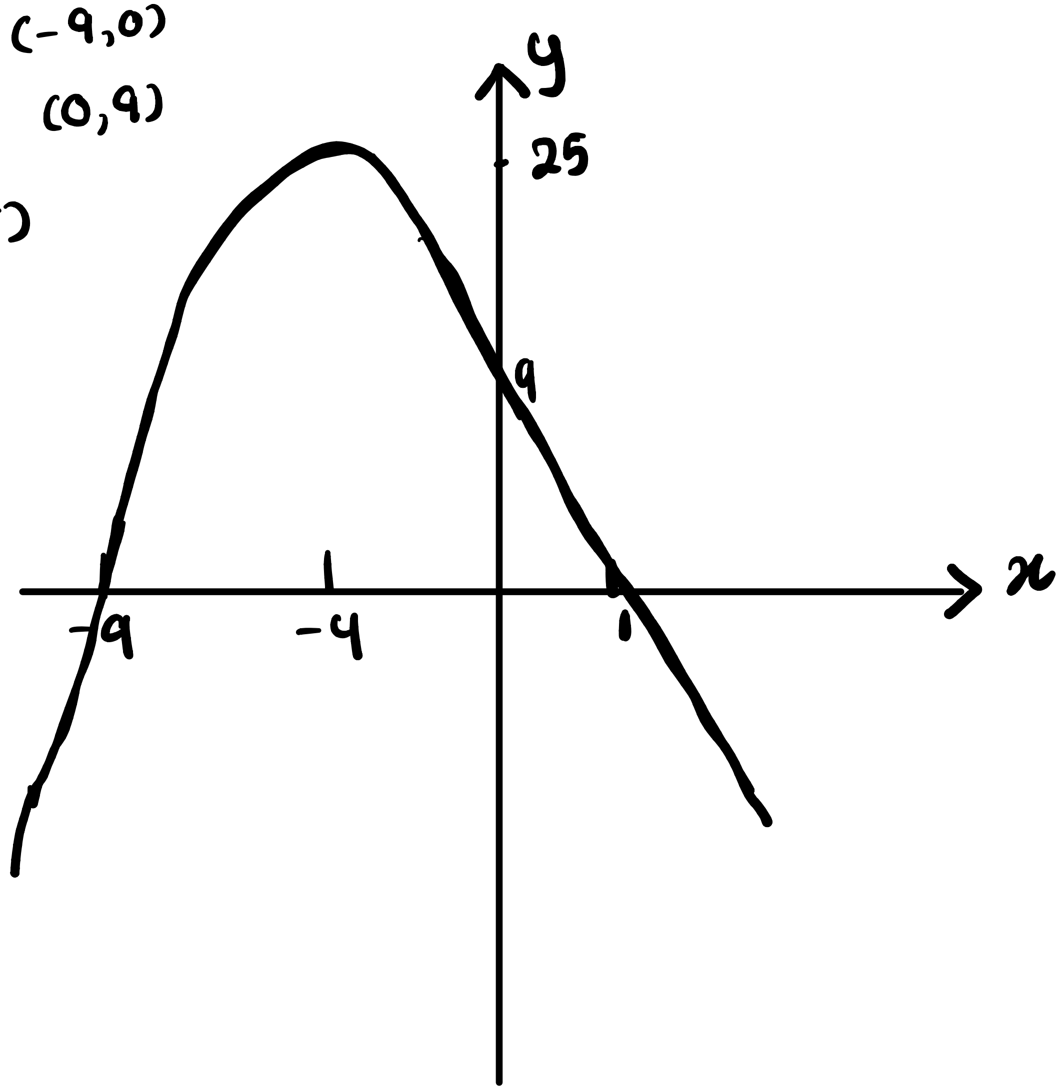
$y=0,\ 9-8x-x^2=0$

$x=1 \quad x=-9$

$(1,0) \quad (-9,0)$

$x=0,\ y=9 \quad (0,9)$

$(-4,25)$



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>0

4. Find the range of values of k for which the equation $kx^2 + k = 8x - 2xk$ has 2 real distinct roots. [4]

$$kx^2 + k + 2kx - 8x = 0$$

$$kx^2 + 2kx - 8x + k = 0$$

$$a = k, \quad b = 2k - 8, \quad c = k$$

$$b^2 - 4ac > 0$$

$$(2k-8)^2 - 4(k)(k) > 0$$

$$\cancel{4k^2} - 32k + 64 - \cancel{4k^2} > 0$$

$$-32k + 64 > 0$$

$$-32k > -64$$

$$k < \frac{64}{32}$$

$$k < 2$$

5. A function f is such that $f(x) = x^2 + 6x + 4$ for $x \geq 0$.

(i) Show that f(x) can be written in the form $(x + a)^2 + b$, where a and b are integers. [2]

$$x^2 + 6x + 4 = x^2 + 2ax + a^2 + b$$

$$2a = 6 \qquad a^2 + b = 4$$

$$a = 3 \qquad 9 + b = 4$$

$$b = -5$$

$$x^2 + 6x + 4 = (x+3)^2 - 5$$

↳ min

$(-3, -5)$

(ii) Write down the range of f. [1]

$$y \geqslant 4$$

(iii) Find f^{-1} and state its domain. [3]

$$y = (x+3)^2 - 5$$

$$x = (y+3)^2 - 5$$

$$x + 5 = (y+3)^2$$

$$\sqrt{x+5} = y + 3$$

$$\sqrt{x+5} - 3 = y$$

$$f^{-1}(x) = \sqrt{x+5} - 3, \ x \geqslant 4$$

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6. (a) Sketch the graph of $y = \left|x^2 - 4x - 12\right|$ showing the coordinates of the point where the graph meets the axes. [3]

$y = x^2 - 4x - 12$

$x = 0,\ y = -12 \quad (0, -12)$

$y = 0,\ x = 6,\ x = -2 \quad (6, 0)\ (-2, 0)$

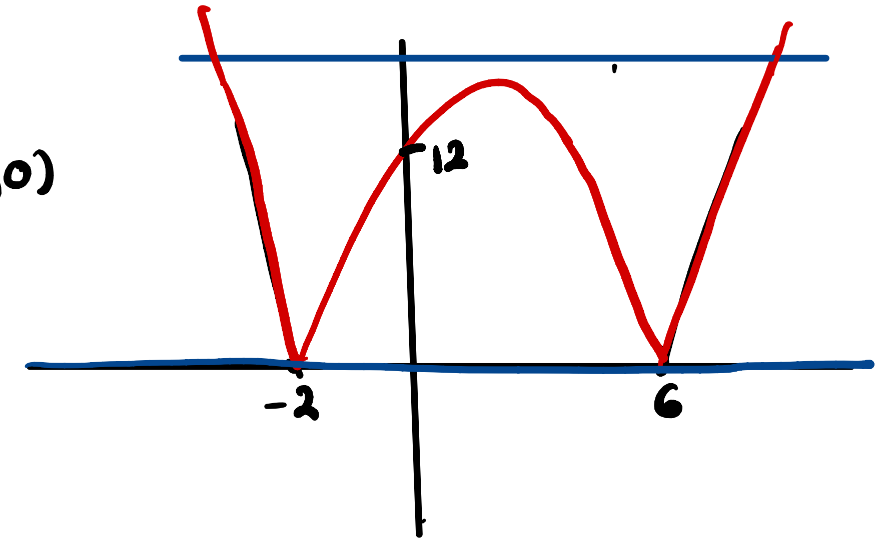
stationary pt:

$x = \frac{6 + (-2)}{2} = \frac{4}{2} = 2$

$y = 4 - 4(2) - 12$

$= 4 - 8 - 12$

$= -16 \quad (2, -16)$



(b) Find the coordinate of the stationary point on the curve $y = \left|x^2 - 4x - 12\right|$. [2]

$(2, 16)$

(c) Find the value of k such that the equation $\left|x^2 - 4x - 12\right| = k$ has only 2 solutions. [2]

$k = 0$ or $k > 16$

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7. Given that $f(x) = 3x^2 + 12x + 2$,

   a. Find the values of a,b and c such that $f(x) = a(x + b)^2 + c$, [3]

$$3x^2+12x+2 = a(x^2+2bx+b^2)+c$$
$$= ax^2+2abx+ab^2+c$$

$a=3$

$2ab=12$

$6b=12$

$b=2$

$ab^2+c=2$

$3\times4+c=2$

$c=2-12$

$=-10$

   b. State the minimum value of f(x) and the value of x at which it occurs, [2]

$x+2=0$

$x=-2$

$3(-2+2)^2-10$

$3\times0-10$

$0-10$

$f(x)=3(x+2)^2-10 \rightarrow (-2,-10)$

minimum value of $f(x)=-10$

value of $x=-2$

   c. Solve $f(\frac{1}{y}) = 0$, giving each answer for y correct to 2 decimal places. [3]

$f(x) = 3(x+2)^2-10$

$3(x+2)^2-10=0$

$3(x+2)^2=10$

$(x+2)^2=\frac{10}{3}$

$x+2=\sqrt{\frac{10}{3}}$ or $x+2=-\sqrt{\frac{10}{3}}$

$x=\sqrt{\frac{10}{3}}-2$ $\quad$ $x=-\sqrt{\frac{10}{3}}-2$

$\frac{1}{y}=\sqrt{\frac{10}{3}}-2$ $\quad$ $\frac{1}{y}=-\sqrt{\frac{10}{3}}-2$

$y=\frac{1}{\sqrt{\frac{10}{3}}-2}=-5.74$ $\quad$ $y=\frac{1}{-\sqrt{\frac{10}{3}}-2}=-0.26$